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AN ITERATIVE SOLUTION PROCEDURE  
FOR CALCULATING POTENTIAL FLOW

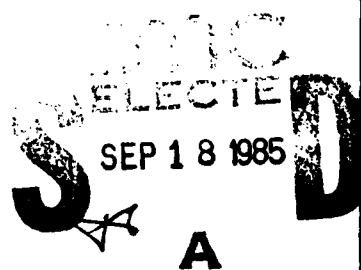
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Yu-Tai Lee and Francis Noblesse

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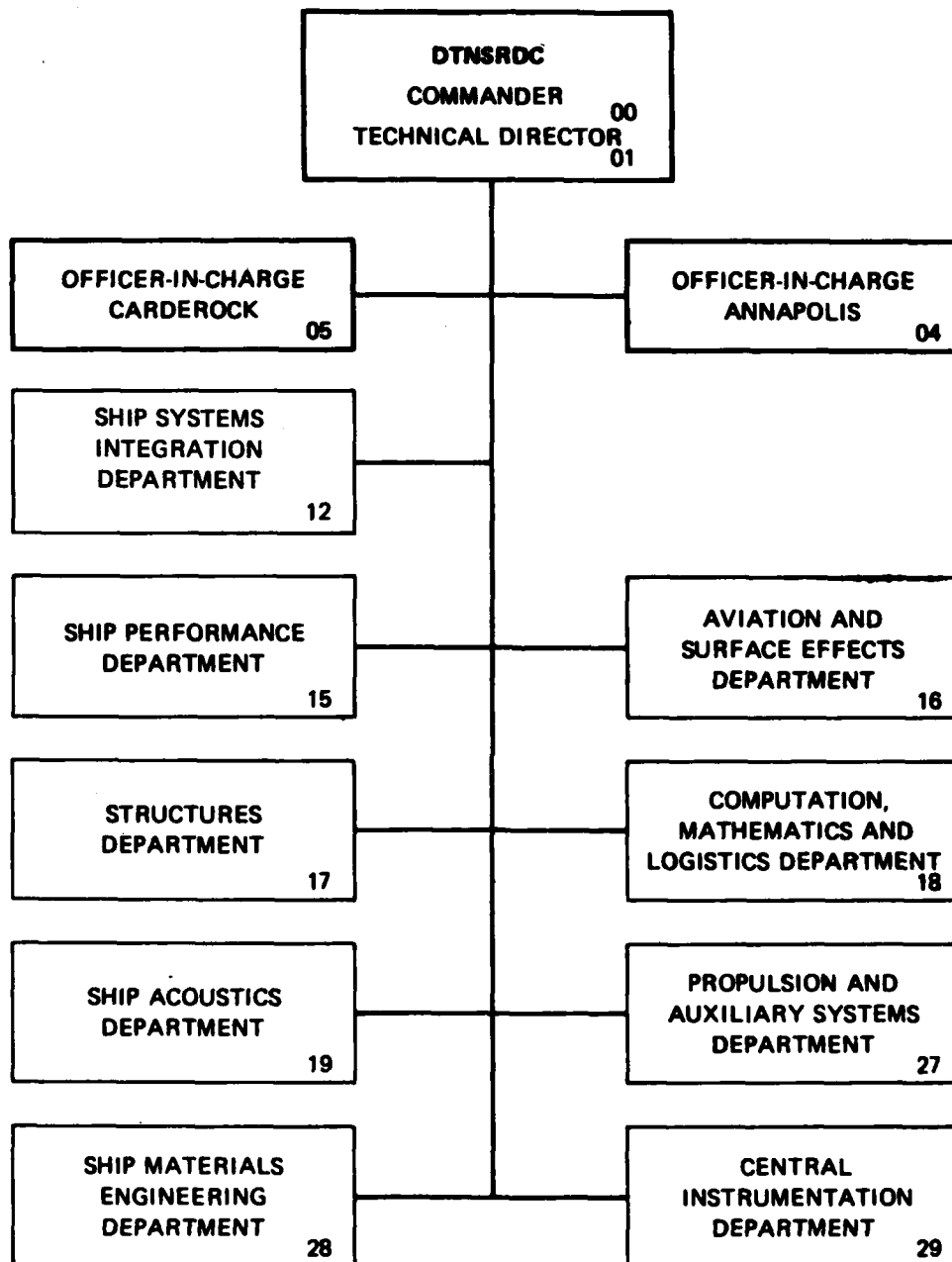
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## ABSTRACT

The report presents the results of a numerical investigation of a novel recurrence relation for iteratively solving modified integral equations for calculating free-surface potential flow. Numerical results are presented for the simple case of wave radiation in the zero- and infinite-frequency limits.

## ADMINISTRATIVE INFORMATION

This research was funded under the David W. Taylor Naval Ship Research and Development Center's Independent Research Program, Program Element 61152N, Task Area ZR0230101, and Work Unit 1843-045.

## INTRODUCTION

Most of the numerical methods reported in the ship-hydrodynamics literature for calculating potential flow about ship-hull forms, whether free-surface effects are taken into account or are neglected as in the so-called double-hull approximation (in which the free surface is treated as a rigid wall), closely follow the pioneering method of Hess and Smith<sup>1</sup>. Briefly, this well-known numerical method consists in determining the density of an auxiliary distribution of sources on the body surface, in the classical manner described by Kellogg<sup>2</sup>, by numerically solving an integral equation. This integral equation is solved by approximating the surface of the body by an ensemble of planar panels and assuming a constant (uniform) source density within each panel. The integral equation is then approximated by a system of linear equations, which is solved by inverting the coefficient-matrix, so-called "matrix of influence coefficients".

A number of variations and extensions of the method of Hess and Smith have been proposed. In particular, alternative integral equations have been

formulated — by using an auxiliary distribution of normal dipoles (instead of sources as in the method of Hess and Smith) or by directly applying Green's third identity to the velocity potential and the fundamental source potential (Green function) — and solved numerically. Important improvements to the basic Hess and Smith method have recently been incorporated into the so-called higher-order panel method<sup>3</sup>, in which curved panels supporting linear-source and quadratic-dipole distributions are used.

An alternative approach for calculating potential flow about a body in an unbounded fluid, in which a modified integral equation for determining the velocity potential is solved iteratively, has been proposed<sup>4,5,6,7</sup>. This alternative approach has also been extended to the calculation of free-surface potential flows<sup>8,9,10</sup>. A main recommendation of this alternative approach is that the modified integral equations upon which the approach is based have regular kernels, whereas the traditional integral equations used in the previously-mentioned methods have singular kernels. Another potential advantage of the alternative approach resides in the use of an iterative solution procedure, which is well suited to the analysis of complex systems requiring a large number of panels<sup>6</sup>. This report presents the results of a limited numerical investigation of the novel recurrence relation proposed in Noblesse<sup>10</sup>. More precisely, this recurrence relation is studied here for the simple case of wave radiation in the zero- and infinite-frequency limits.

## INTEGRAL EQUATIONS

This study considers potential flow due to translation or rotation of a rigid, nonlifting body through an incompressible and inviscid fluid. The surface of the body is denoted by  $S$ , and  $\hat{n}$  represents the unit normal vector to  $S$  pointing inside the body. Furthermore,  $\hat{x}(x, y, z)$  and  $\hat{\xi}(\xi, \eta, \zeta)$  represent the position vectors, with respect to a rectangular system of coordinates



attached to the moving body, of any two points on the body surface  $S$ . The coordinates  $\vec{x}$  and  $\vec{\xi}$  are adimensional, with respect to some length characterizing the size of the body taken as reference. The velocity potential,  $\phi$  say, of the flow likewise is adimensional, with the body characteristic length and some characteristic speed of its motion used as reference. The normal component of the fluid velocity at any point  $\vec{x}$  on the body surface, given by  $\phi_n = \nabla\phi \cdot \vec{n}$ , is equal to the normal component of the velocity of  $S$  at  $\vec{x}$ , and thus is known for all six fundamental motions of the body (translation and rotation about the  $x$ ,  $y$ ,  $z$  axes).

The Green function appropriate for an unbounded fluid is given by  $4\pi G(\vec{\xi}, \vec{x}) = -1/r$  where  $r = [(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2]^{1/2}$ . Application of Green's third identity to the velocity potential  $\phi(\vec{x})$  and the foregoing free-space Green function  $G(\vec{\xi}, \vec{x})$  yields the following classical integral equation for determining the potential  $\phi(\vec{\xi})$  at any point  $\vec{\xi}$  on  $S$ :

$$\phi(\vec{\xi})/2 = f(\vec{\xi}) + L(\vec{\xi}; \phi), \quad (1)$$

where  $f(\vec{\xi})$  is the known (since  $\phi_n$  is specified on  $S$ ) potential defined as

$$f(\vec{\xi}) = -\int_S G(\vec{\xi}, \vec{x}) \phi_n(\vec{x}) da(\vec{x}), \quad (2)$$

and  $L(\vec{\xi}; \phi)$  represents the integral transform of  $\phi$  defined as

$$L(\vec{\xi}; \phi) = \int_S G_n(\vec{\xi}, \vec{x}) \phi(\vec{x}) da(\vec{x}). \quad (3)$$

In equations (2) and (3),  $da(\vec{x})$  represents the differential element of area of the surface  $S$  at the point  $\vec{x}$ .

The previously-mentioned modified integral equation corresponding to the classical integral equation (1) takes the form<sup>5,6,7</sup>

$$\phi(\vec{\xi}) = f(\vec{\xi}) + L'(\vec{\xi}; \phi), \quad (4)$$

where  $f(\vec{\xi})$  is the known source potential given by equation (2) and  $L'(\vec{\xi};\phi)$  is the integral transform of  $\phi$  defined as

$$L'(\vec{\xi};\phi) = \int_S G_n(\vec{\xi},\vec{x})[\phi(\vec{x})-\phi(\vec{\xi})]da(\vec{x}). \quad (5)$$

A noteworthy feature of the modified integral equation (4) is that the integrand  $G_n(\vec{\xi},\vec{x})[\phi(\vec{x})-\phi(\vec{\xi})]$  of the integral (5) remains finite as the "integration point"  $\vec{x}$  approaches the "calculation point"  $\vec{\xi}$ .

The radiation potential for a body intersecting a free surface must satisfy the free-surface boundary condition  $\partial\phi/\partial z - (\omega^2 L/g)\phi = 0$  on the equilibrium plane of the free surface taken as the plane  $z = 0$  with the  $z$  axis pointing upward, where  $\omega$  = radiant frequency of oscillatory motion of the body,  $L$  = body characteristic length and  $g$  = acceleration of gravity. In the zero-frequency limit  $\omega^2 L/g = 0$  and the infinite-frequency limit  $\omega^2 L/g = \infty$ , the free-surface boundary condition takes the simple degenerate forms  $\partial\phi/\partial z = 0$  and  $\phi = 0$ , respectively. The Green functions  $G^0$  and  $G^\infty$  corresponding to these limiting cases are given by  $4\pi G^0 = -1/r - 1/r'$  and  $4\pi G^\infty = -1/r + 1/r'$ , where  $r = [(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2]^{1/2}$  as was defined previously and  $r' = [(\xi-x)^2 + (\eta-y)^2 + (\zeta+z)^2]^{1/2}$ .

The modified integral equation, corresponding to equation (4), for the zero-frequency potential,  $\phi^0$  say, takes the form<sup>1)</sup>

$$\phi^0(\vec{\xi}) = f^0(\vec{\xi}) + L^0(\vec{\xi};\phi^0), \quad (6)$$

where the potentials  $f^0(\vec{\xi})$  and  $L^0(\vec{\xi};\phi^0)$  are defined as

$$f^0(\vec{\xi}) = -\int_S G^0(\vec{\xi},\vec{x})\phi_n^0(\vec{x})da(\vec{x}),$$

$$L^0(\vec{\xi};\phi^0) = \int_S G^0_n(\vec{\xi},\vec{x})[\phi^0(\vec{x})-\phi^0(\vec{\xi})]da(\vec{x}).$$

The corresponding modified integral equation for the infinite-frequency potential,  $\phi^\infty$  say, takes the form<sup>11</sup>

$$[1-w(\vec{\xi})]\phi^\infty(\vec{\xi}) = f^\infty(\vec{\xi}) + L^\infty(\vec{\xi}; \phi^\infty), \quad (7)$$

where the potentials  $f^\infty(\vec{\xi})$  and  $L^\infty(\vec{\xi}; \phi^\infty)$  are defined as

$$f^\infty(\vec{\xi}) = -\int_S G^\infty(\vec{\xi}, \vec{x}) \phi_{,n}^\infty(\vec{x}) da(\vec{x}),$$

$$L^\infty(\vec{\xi}; \phi^\infty) = \int_S G^\infty_n(\vec{\xi}, \vec{x}) [\phi^\infty(\vec{x}) - \phi^\infty(\vec{\xi})] da(\vec{x}),$$

and  $w(\vec{\xi})$  is the waterplane integral defined as

$$2\pi w(\vec{\xi}) = -\zeta \iint_\sigma [(\xi-x)^2 + (\eta-y)^2 + \zeta^2]^{-3/2} dx dy,$$

with  $\sigma$  = portion of the mean free-surface plane inside the body ( $w \equiv 0$  for a fully-submerged body). By expressing the integrand of the foregoing waterplane integral in the form  $\partial(x-\xi)[(y-\eta)^2 + \zeta^2]^{-1}[(x-\xi)^2 + (y-\eta)^2 + \zeta^2]^{-1/2}/\partial x$  and using Green's theorem, we may express the waterplane integral in the form of the equivalent waterline integral

$$2\pi w(\vec{\xi}) = -\zeta \int_C (x-\xi)[(y-\eta)^2 + \zeta^2]^{-1}[(x-\xi)^2 + (y-\eta)^2 + \zeta^2]^{-1/2} dy,$$

where  $c$  represents the intersection curve of the body surface  $S$  with the plane  $z = 0$ .

#### FLOW ABOUT AN ELLIPSOID IN AN UNBOUNDED FLUID

The practicality of solving the classical integral equation (1) by using an iterative solution procedure based on the Neumann series

$$\phi^{(n+1)}(\vec{\xi})/2 = f(\vec{\xi}) + L(\vec{\xi}; \phi^{(n)}), \quad (8)$$

with  $n \geq 0$  and  $\phi^{(0)} \equiv 0$ , has been investigated numerically for the simple case of flow about an ellipsoid with major, intermediate and minor dimensions equal to

1, 0.15 and 0.1, respectively. Figure 1 depicts the relative errors associated with the  $n^{\text{th}}$  approximations  $A_{ij}^{(n)}$  to the added-mass coefficients  $A_{ij}$  for all six fundamental motions of the ellipsoid ( $i = 1$  to 6). This figure shows that the Neumann series (8) converges fairly rapidly for all motions except for surge (translation in the direction of the major axis). This numerical result is in agreement with the theoretical result obtained in Noblesse and Triantafyllou<sup>7</sup> for translatory motions of ellipsoids. In particular, it is proved in this study that the Neumann series (8) fails to converge for a needle-like spheroid in longitudinal translation. Figure 1 shows that the sixth and fourth iterations are essentially exact for sway and heave (translations in the directions of the intermediate and minor axes), respectively. Convergence is even faster for roll and pitch, for which three iterations are sufficient. Finally, five iterations are necessary for yaw.

The velocity potential  $\psi(\vec{\xi})$  associated with the modified integral equation (4) and defined as

$$\psi(\vec{\xi}) = f^2(\vec{\xi}) / [f(\vec{\xi}) - L^*(\vec{\xi}; f)]$$

is proved in [7] to be exact for translatory motions of any ellipsoid. This theoretical result has been verified numerically for the ellipsoid considered in Figure 1. The potential  $\psi(\vec{\xi})$  has also been verified numerically to be exact for rotational motions (roll, pitch and yaw). The accuracy of the calculated added mass coefficients for all six fundamental motions is within 0.4% by using approximately 400 panels. The above-defined potential  $\psi(\vec{\xi})$  corresponds to the first approximation  $\phi^{(1)}(\vec{\xi})$  in the sequence of iterative approximations  $\phi^{(n)}(\vec{\xi})$  defined by the recurrence relation<sup>11</sup>

$$\phi^{(n+1)}(\vec{\xi}) = \phi^{(n)}(\vec{\xi}) f(\vec{\xi}) / [\phi^{(n)}(\vec{\xi}) - L^*(\vec{\xi}; \phi^{(n)})],$$

with  $n \geq 0$  and  $\phi^{(0)}(\vec{\xi}) \equiv f(\vec{\xi})$ , associated with the modified integral equation (4). The practicality of using this recurrence relation for solving the

modified integral equations (6) and (7) is tested numerically in the following section for a typical hull form.

#### FLOW ABOUT A SHIP FORM AT ZERO AND INFINITE FREQUENCY

The foregoing recurrence relation for potential flow in an unbounded fluid takes the form

$$\phi_0^{(n+1)}(\vec{\xi}) = \phi_0^{(n)}(\vec{\xi}) f_0(\vec{\xi}) / [\phi_0^{(n)}(\vec{\xi}) - L_0(\vec{\xi}; \phi_0^{(n)})] \quad (9)$$

for flow in the zero-frequency limit, corresponding to the modified integral equation (6). The corresponding recurrence relation associated with the modified integral equation (7) for flow in the infinite-frequency limit is

$$\phi_\infty^{(n+1)}(\vec{\xi}) = \phi_\infty^{(n)}(\vec{\xi}) f_\infty(\vec{\xi}) / [\{1 - w(\vec{\xi})\} \phi_\infty^{(n)}(\vec{\xi}) - L_\infty(\vec{\xi}; \phi_\infty^{(n)})] \quad (10)$$

The recurrence relations (9) and (10), with  $n > 0$  and  $\phi_0(0) = f_0$ ,  $\phi_\infty(0) = f_\infty$ , have been used for calculating the six added-mass coefficients  $A_{1i}$  for  $i = 1$  to 6 in the zero- and infinite-frequency limits for the ship form depicted in Figure 2, with beam/length and draft/length ratios equal to 0.15 and 0.05, respectively. The relative errors associated with the  $n^{\text{th}}$  iterative approximations  $A_{1i}^{(n)}$  to the added mass coefficients  $A_{1i}$  for all six fundamental motions of the ship ( $i = 1$  to 6) are depicted in Figures 3 and 4 for flow in the zero- and infinite-frequency limits, respectively. These figures show that six to eight iterations are required.

#### CONCLUSION

The numerical results presented in this report indicate that the recurrence relations for free-surface flows<sup>10</sup> corresponding to the recurrence

relations (9) and (10) for flow in the zero- and infinite-frequency limits may provide an efficient solution procedure compared to that associated with the classical integral equation as shown in Figure 1, especially for cases (hull form equipped with a bulb or a sonar dome, low Froude number, high frequency) for which a large number of panels is required<sup>6</sup>. Further numerical study of these recurrence relations is required for ascertaining their practical usefulness.

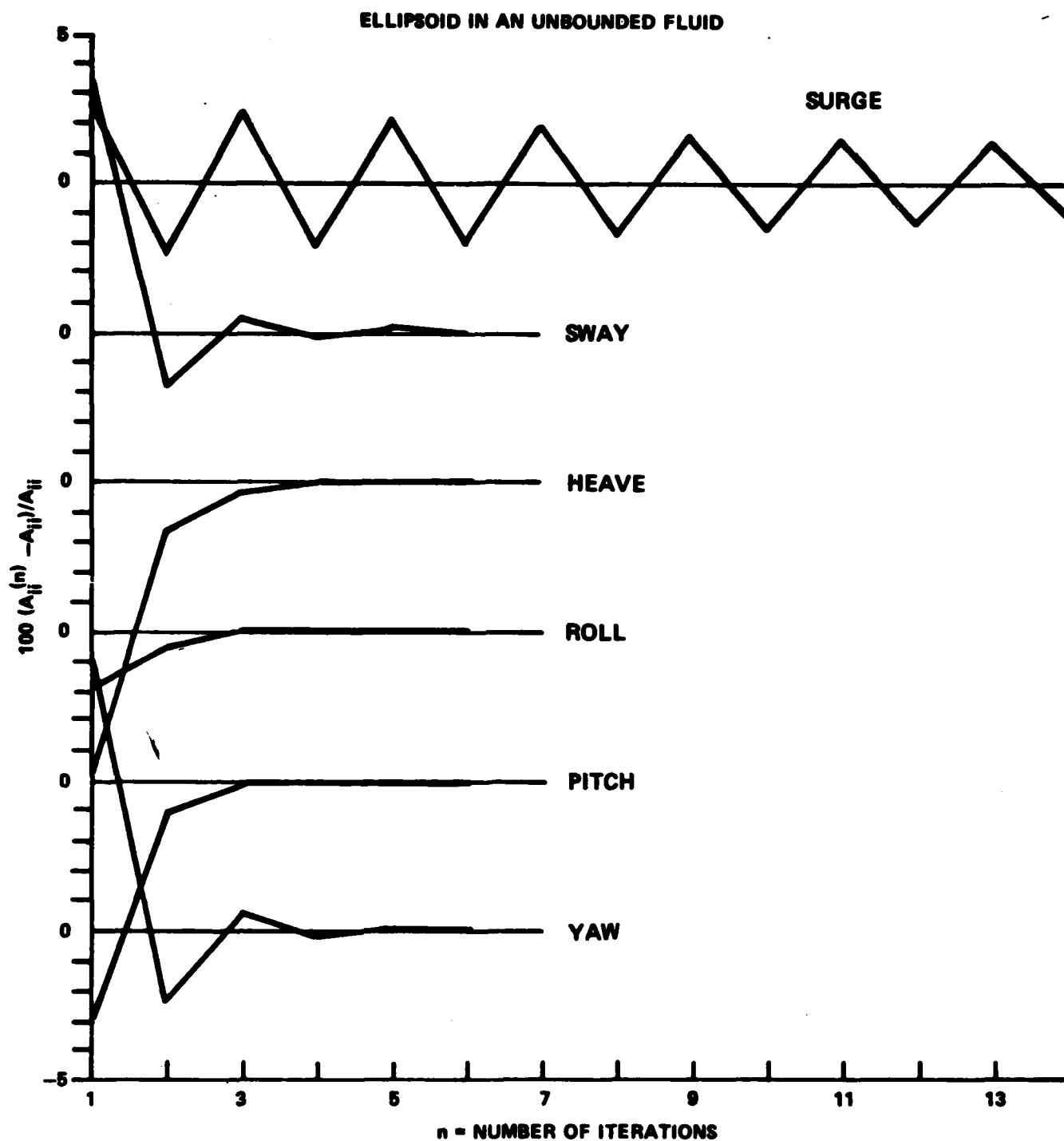


Figure 1 - Convergence of the Neumann Series (8) Associated with the Integral Equation (1) for Flow About An Ellipsoid

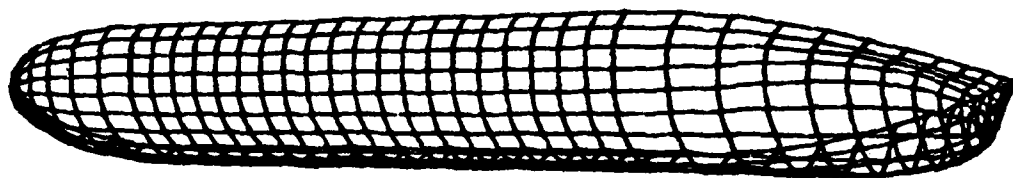


Figure 2 - Hull Form Used in Calculations Presented in  
Figures 3 and 4



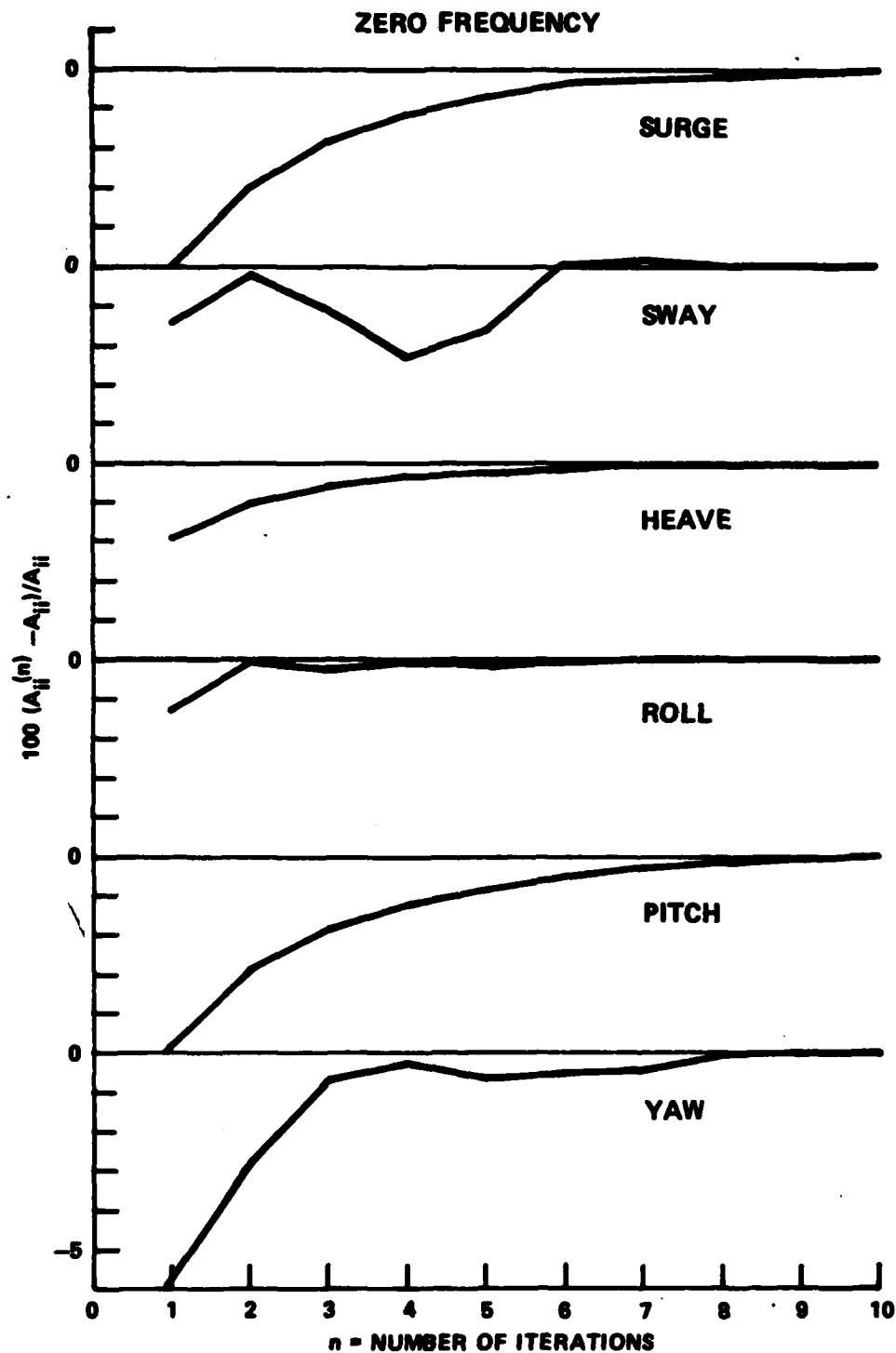


Figure 3 - Convergence of the Recurrence Relation (9) Associated with the Integral Equation (6) for Zero-Frequency Flow About a Typical Ship Form

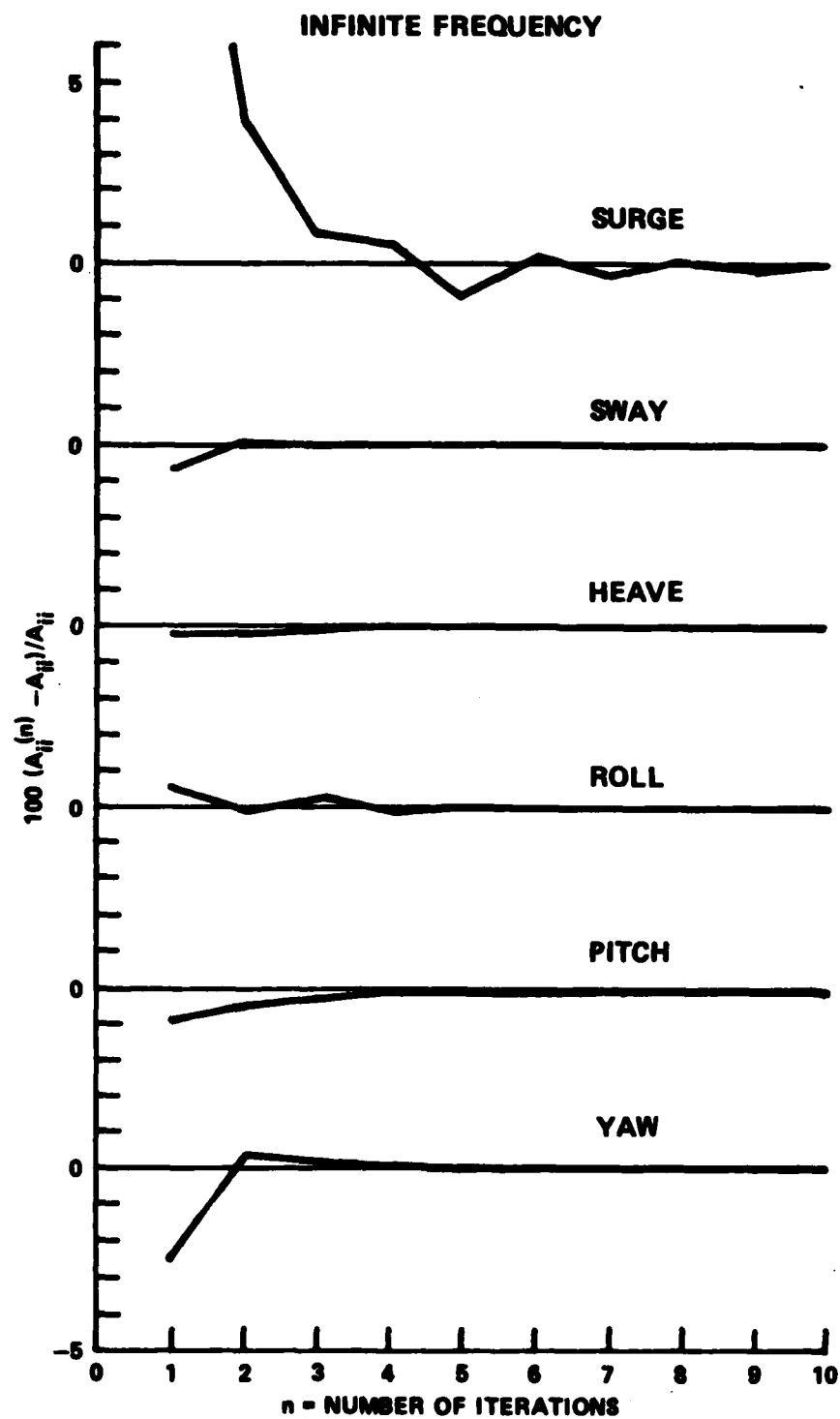


Figure 4 - Convergence of the Recurrence Relation (10) Associated with the Integral Equation (7) for Infinite Frequency Flow About a Typical Ship Form

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